CHARACTERISTICS OF THE SPATIAL MULTIFRACTAL PROPERTIES OF HOURLY SPATIAL RAINFALL OVER JAPAN

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INTRODUCTION

There is an urgent need of high resolution meteorological information to cater for the new focus on the hydrological response at small spatial and temporal scales, which has arisen largely due to the problems related to urbanization. Alternative means of estimating precipitation at small scales are beneficial, as high resolution data acquisition is a time consuming and expensive task. Successful spatial and temporal downscaling methods find many uses in the process of solving hydrological problems, including, the deriving of high resolution synthetic data based on low resolution observations, interpolation of the measurements of low density rain gauge networks, downscaling of the output of regional climatic models and global circulation models.

Fractal theory is a useful tool in establishing scale relationships of geophysical fields, including rainfall in space and time. There have been a number of reports on rainfall downscaling models that are based on multifractals – a special branch of fractal theory. (Olsson 1996, Over and Gupta 1994, Pathirana et al. 2001.) To apply multifractals successfully in solving practical problems related to downscaling, it is important to understand the existence, nature and properties of multifractal properties in the rainfall.

In this paper, a multifractal analysis in performed on spatial rainfall data obtained by interpolating the recordings of a rain gauge network. First, the rain gauge network itself was examined, in order to assess its spatial homogeneity and to understand the limits of the spatial resolutions that can be used in analysis. Then the resulting spatial rainfall snapshots were analyzed for multifractal properties. The derived multifractal model parameters were examined for their properties including the seasonal behavior.

DATA PROCESSING

Gauge measured hourly rainfall data for the year 1997, of the AMeDAS rain-gauge network of the Japan Meteorological Agency (figure 1), was used as source data. There are a number of techniques used in practice to interpolate point-scale data in to a spatial distribution: Thiessen polygons, nearest neighborhood method, Krigging, to name a few. However, one of the major assumptions in many of these methods is that the rain fields have smooth variation in space – a fact which is disputed by observation and is contradictory to the basic premise of multifractal models: the discontinuity. Though, the use of an interpolation method is inevitable in processing point scale data in to spatial data, care should be taken to establish realistic limits to the resolutions that can be reached. Another important concern is the homogeneity of the point data network.

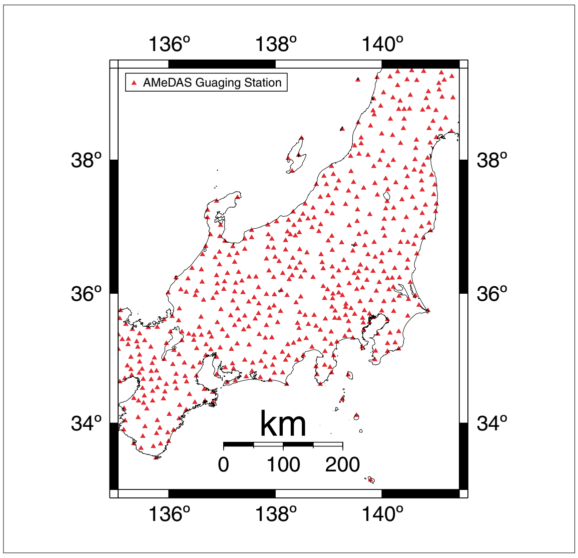

Fig. 1: The area used for multifractal analysis. The bounds are 135.05W, 141.45E, 39.35N and 32.95S.

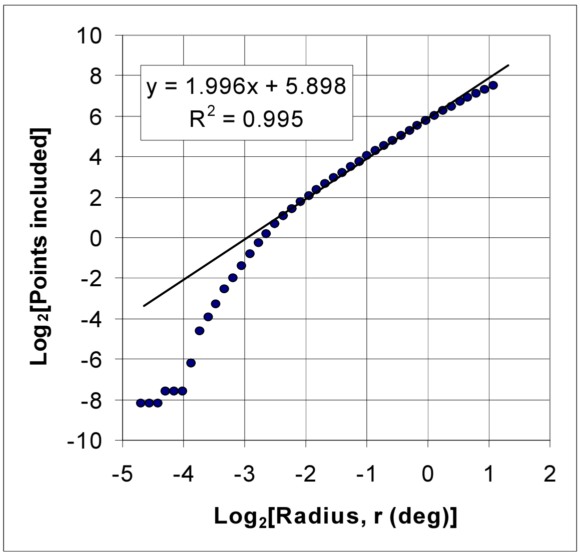

Fig. 2: The plot of $\langle C(r)\rangle$ against r, to estimate D_c. (Logarithms of base 2 are used.)

If there is a serious heterogeneity of the measurement stations, corrections should be done at processing (Lovejoy and Schertzer 1994).

The correlation dimension analysis is an excellent method to investigate a network of measurement points for above problems. The correlation dimension D_c is defined as follows:

$$\langle C(r)\rangle = \frac{1}{N}\sum_{i}^{S}\sum_{j}^{S}\rho_{(i,j),i\neq j}, \quad \rho_{(i,j)} = \begin{cases} 1 & \text{for } |\vec{r}_i - \vec{r}_j| \leq r \\ 0 & \text{for } |\vec{r}_i - \vec{r}_j| > r \end{cases}$$

where N is the number of elements in the point set S and $\vec{r}$ is the positional vector of an element and r is distance, then if a fractal set is scaling,

$$\langle C(r)\rangle \propto r^{D_c}. \tag{1}$$

Figure 2 shows the variation of the quantity $\langle C(r)\rangle$ with the radius, r. There are a number of points of importance, shown by this figure. First, the slope of the straight line fitting to a part of the range of scales analyzed, is practically equal to 2.0 – the embedding dimension of the point set. This indicates that it is possible to consider that the network in homogeneous in that range of scale. The second point of importance is the limits of the range of scales itself. The upper limit of this range practically exceeds the largest scale analyzed (2.0^0). The lower limit is around 0.1^0. The lower scale break indicates the smallest scale where the rain gauge network can reliably represent a spatial quantity. Thus, this imposes an important limitation on further analysis, namely, the lower limit of accurate estimation of spatial properties. While, the upper scale limit seems to be much larger than the investigated scales, there is another factor that restricts the largest upper scale than can be analyzed, namely, the decrease of reliability of moment estimates, due to the limited number of samples. Hence, only the scale range 0.1^0-0.8^0 was subjected to further analysis.

MULTIFRACTAL MODEL

The universal multifractal model (Tessier et al. 1993) proposes that the q^{th} statistical moment of a field R, shows the following scaling behavior:

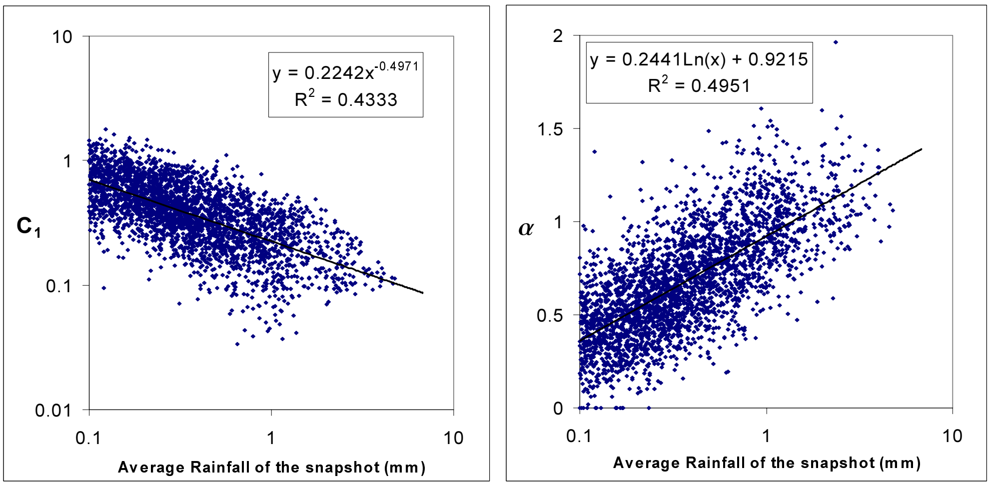

Fig. 3: Both C_1 and α shows a strong relationship with the mean value of the rainfall snapshot.

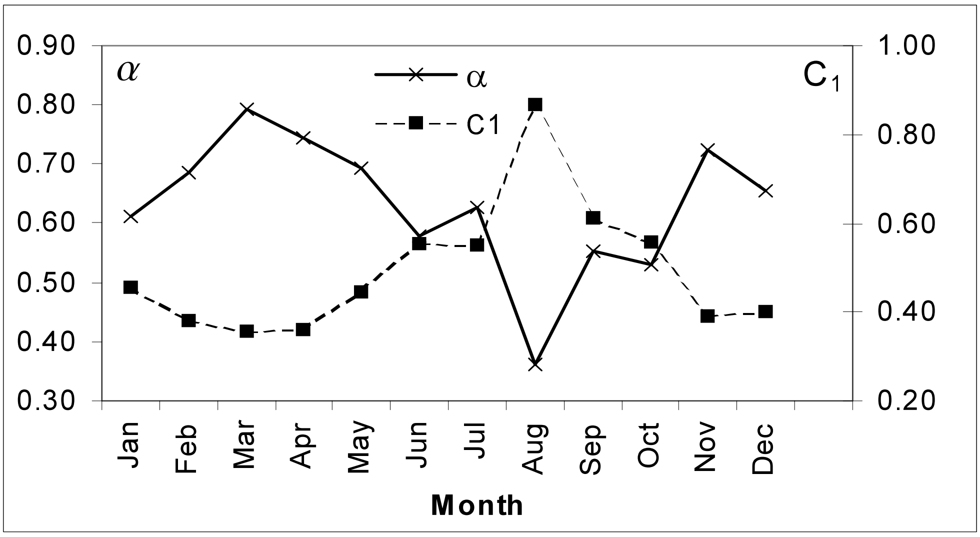

Fig. 4: Monthly average values of C_1 and α in the year of 1997. There is a marked anomaly of the values in the month of August.

$$E[R_\lambda^q] \propto \lambda^{K(q)}, \text{ where } \quad K(q) = \begin{cases} \frac{C_1}{\alpha - 1}\left(q^\alpha - q\right) & \alpha \neq 1 \\ C_1 q \log\left(q\right) & \alpha = 1 \end{cases} \quad (\text{for } 0 \leq \alpha \geq 2,\ q \geq= 0) \tag{2}$$

where λ is the non-dimensional scale ratio. According to this model, the scaling behavior can be quantified with two parameters, namely, C_1 and α. From equation 2, a scaling behavior of a rainfall field can be verified by plotting $E[R_\lambda^q]$ against λ in a double logarithmic paper. If the variation of $\log[E[R_\lambda^q]]$ with λ is linear for each value of q, the field can be considered scaling. After the scaling properties of the field was verified (the results are not produced for brevity), the estimation of the multifractal parameters, C_1 and α, was done. The Double Trace Moment (DTM) technique proposed by Lavallée (1991) was used for this purpose. (See Tessier et al. 1993, also.)

RESULTS

The average multifractal parameter values were $\alpha = 0.60 \pm 0.31$ and $C_1 = 0.53 \pm 0.34$. They showed a strong relationship with the grid average rainfall or the large-scale forcing. (Figure 3).

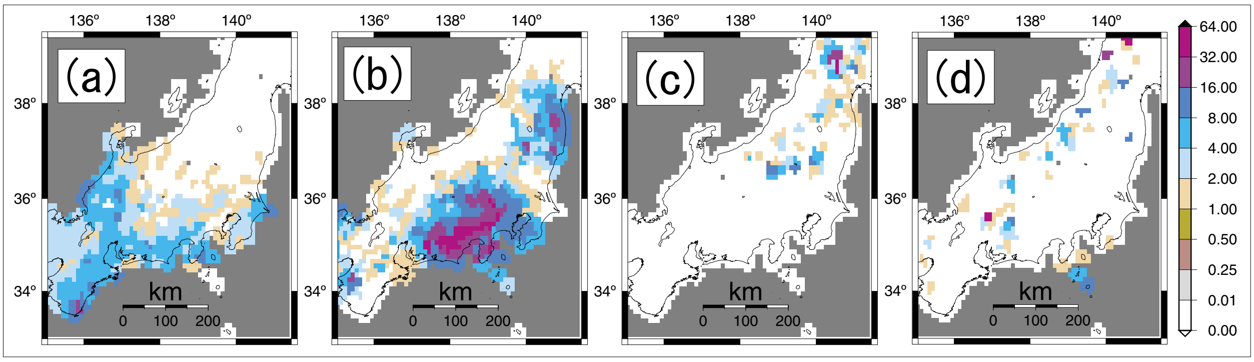

Fig. 5: Some examples of rainfall events that are corresponding to extreme values of C_1 and α:

Class	Time	α	C_1	Class	Time	α	C_1
I	(a) 1997-Apr-06 13:00	1.409	0.092	II	(c) 1997-Aug-03 20:00	0.555	0.583
	(b) 1997-Jun-20 12:00	1.061	0.114		(d) 1997-Jul-29 16:00	0.322	1.099

The coefficients of correlations for α and C_1 were 0.62 and -0.51, respectively. The investigation of the seasonal variation of the parameters showed a marked anomaly in the midsummer (figure 4).

DISCUSSION

Since the scaling exponent, $k(q)$ in equation 2 is related to the statistical moment by a power law, the value of the exponent does not directly depend on the average value of the field being analyzed. Further, the usual practice in multifractal data analysis is to normalize the data to have unit mean. Thus, only properties which affect the values of multifractal constants should be the intensity distribution (relative magnitudes) and the patterns in the rainfall.

The seasonal anomaly of multifractal parameters should be due to the seasonal variation rainfall patterns. In order to examine this relationship, a number of rainfall snapshots that generated extreme parameters, were examined. These revealed that there is a clear relationship between the spatial patterns of rainfall and the parameter values. It was found that the rainfall resulting from large disturbances result in larger α (and smaller C_1) values, while, scattered rainfall due to local thunderstorms give smaller α (and larger C_1) values. Figure 5 shows two examples each of these two categories of rainfall.

The analysis of spatial distribution shows that the multifractal models can be used to represent statistical distribution of rainfall and that multifractal parameters can be used to classify rainfall in to different patterns.

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